Development of a new method for solving the wave equation, DOWT (Discrete Operational Wave Theory)

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Background

Establishment of the ACROSS (Accurately Controlled, Routinely Operated, Signal System), which is an observation technology in geophysical exploration and enables us to acquire very accurate data in frequency domain urged us to develop a theoretical support for wavefield analysis which is capable of solving the wave equation representing a large body with the most general structures in frequency domain. Here, we propose a new method for solving the wave equation, DOWT (Discrete Operational Wave Theory).

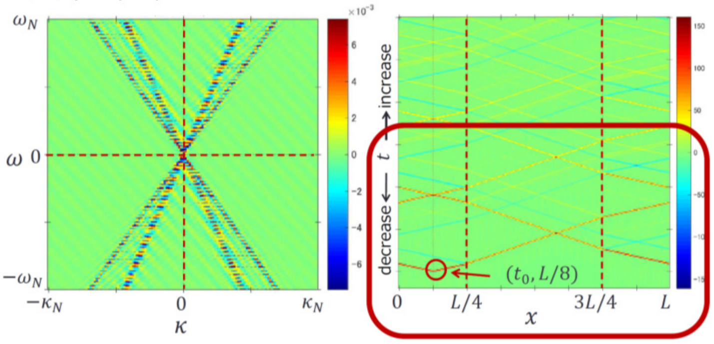
Features of DOWT (Discrete Operational Wave Theory)

- Boundary conditions representing discontinuity of material properties are thoroughly eliminated.
  - The discontinuity is expressed in the Generalized Polynomial Expansion using generalized functions such as the Dirac delta function and the Heaviside step function.
- The wave equation is expressed in the frequency-wavenumber domains and solved as linear equations.
  - All functions which appear in the wave equation are expressed in Fourier series expansions.
- The algorithm in simple.
- Applicable to both the elastic wave equation and Maxwell's equations.

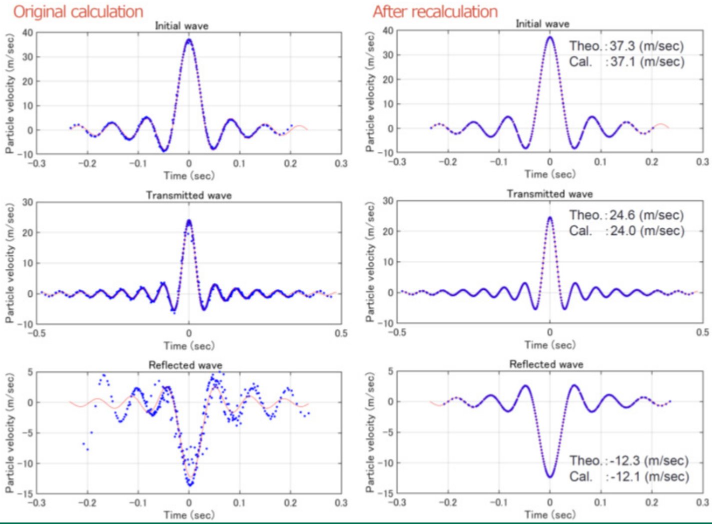
1C1D calculation of the elastic wave equation: Inhomogeneous case

Dispersion relation of particle velocity $v(t,x)$ (real part)

Travel time curve of $v(t,x)$



Reconstruction of waveforms



Generalized Polynomial Expansion

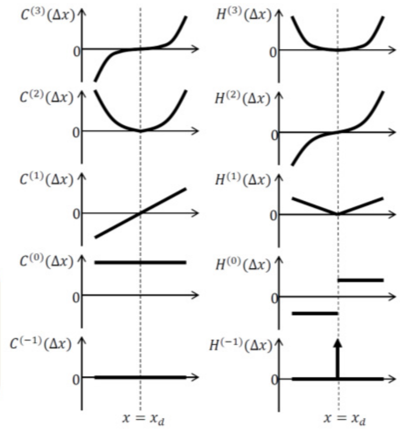
- Basis function of continuous part

$$C^{(j)}(\Delta x) = \frac{\Delta x^j}{n!}, \Delta x = x - x_d$$

- Basis function of discontinuous part

$$H^{(j)}(\Delta x) = C^{(j)}(\Delta x) H(\Delta x)$$
$$H^{(-1)}(\Delta x) = \delta(\Delta x)$$

$$y(x) = y_{cont}(x) + y_{disc}(x)$$
$$y_{cont}(x) = a_0(x) C^{(0)}(\Delta x)$$
$$y_{disc}(x) = \sum_d \{ b_{-1}(x_d) H^{(-1)}(\Delta x) + b_0(x_d) H^{(0)}(\Delta x) \}$$



Travel time

A→C (C')→D:

$\left(\frac{L}{8} + \frac{L}{4}\right) \sqrt{\rho_1 / c_1} =$ 0.315(sec)

From the cal. 0.317(sec)

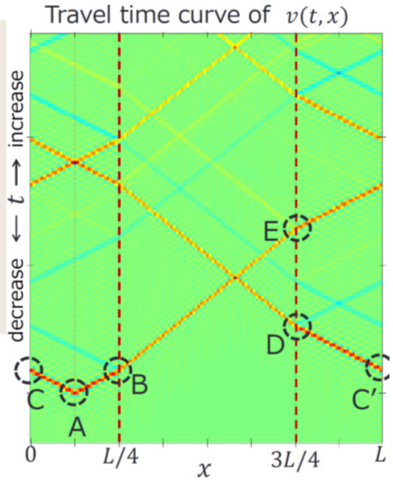
B→E:

$L/2 \sqrt{\rho_2 / c_2} =$ 0.750 (sec)

From the cal. 0.749 (sec)

Good agreement!

Travel time curve of $v(t,x)$



Analytic solution

	Eigenfrequency (Hz)
1	0.752
2	0.853
3	1.520
4	1.684
5	2.316
6	2.480
7	3.147
8	3.248

Good agreement!

Power spectrum of $v(\omega, \kappa)$